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CULTURAL COMMENTARY

What Hath Rubik Wrought?

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In May, 1980 the Ideal Toy Company launched its newest offering, *Rubik's Cube,* at a party in Hollywood, hosted by Zsa-Zsa Gabor and Solomon W. Golomb. Of course Gabor, like the cube, is a Hungarian product but who is Golomb? Well, he is a mathematician at the University of Southern California and an expert in number theory, combinatorics, abstract algebra and coding theory.

Does the conjunction of a movie star and a mathematician seem strange? Can mathematics entertain and can play be serious? Indeed it can and the cube is only the latest (and possibly the best) manifestation of recreational mathematics.

A cavil. Let me say that one does not need to know mathematics in order to solve the cube or the appreciate it as a beautifully constructed mechanism. Nor does knowing the mathematics of the cube necessarily make one a cube virtuoso, able to restore the cube in less than a minute.

However, the cube moves as it does and the small cubelets can occupy just the spaces they do according to a precise set of rules governing patterns and their rearragements. The mathematics of pattern rearrangement or permutations of objects is called *group theory,* a branch of abstract algebra.

Rubik invented the cube as an aid in teaching his students three-dimensional thinking. In that effort he was marvelously successful. But now the cube has become the darling of algebraists, who use it to teach group theory to their students.

Curiously, previous generations toyed with the device which is a direct precursor of Rubik's cube, and mathematicians have seized upon it as well. This was an invention of America's greatest puzzlist, Samuel Loyd, who produced and object in 1878 called the *15 puzzle.* It is a four-by-four tray holding fifteen one-by-one squares (numbered 1, 2, . . . , 15) that are grooved to allow any one of them to slide past an adjacent square and into an empty space. The space can be thought of as an imaginary sixteenth square. The puzzle requires that one reestablish the usual serial order among the numbered squares after an initial jumbled order has been imposed. The puzzle is still available worldwide, in various forms, and, I daresay, should persist along with the cube down all the generations to come.

Let me note some comparisons.

Both Rubik's cube and Loyd's puzzle are ingeniously constructed. This gives the mechanisms themselves a beauty and harmony that appeal both to our eye and to our intellect.

The *15 puzzle* had a vogue as great as the cube does today, especially in Europe. Journalists of the late nineteenth century reported that the puzzle created headaches and neuroses. Today, many of us (cubic rubes?) can report similar effects from our attempts at conquering the cube.

Both the cube and the puzzle have a mathematical description rooted in group theory. To master the toy in each case is akin to solving the underlying mathematical problem.

It's worthwhile to note that the inventions of group theory, Loyd's puzzle and Rubik's cube, occurred approximately one hundred years apart. The origins of group theory are usually traced to 1770 and the ideas of the great French mathematician Joseph Louis Lagrange. A century later, Lagrange's initial study of permutations had become a full-blown branch of mathematics: group theory. So by 1878, when Loyd produced the *15 puzzle*, the mathematics was available to analyze it, which was done in an article in the fledgling *American Journal of Mathematics* (Volume 2, 1879).

Now the exposition of the mathematics of the cube is more complicated but still a task of group theory. Instead of numbered squares rearranged by sliding in a two-dimensional tray we have cubelets being interchanged by rotating in three-space. (A good discussion of this, accessible to the non-mathematician, is in Douglas Hofstadter's column, *Metamagical Themas,* in *Scientific American,* March 1981).

The interplay between group theory and games in general and group theorists and the cube in particular continues unabated at this writing. Let me cite one important example.

John Horton Conway, at the University of Cambridge, England, is considered to be a mathematician of the highest rank and has made significant contributions to group theory. In 1969, he discovered three so-called "sporadic simple groups" which helped to complete the largest single research effort in mathematics; the search for all finite simple groups. This work began in 1870 and ended in 1981. The final synthesis of his work, which is now going on, must be extracted from the equivalent of a research paper some five-thousand pages long. Conway is also the inventor of numerous (one can actually say infinitely many) wonderful games. He has recently co-authored *winning ways (*Academic Press, London, 1982) which is expected to become the standard work on combinatorial games well through the twenty-first century.

As to the cube, Conway has not only mastered it, but invented a variation of the usual play called — "three looks." Here the player inspects the cube, then holds it under a

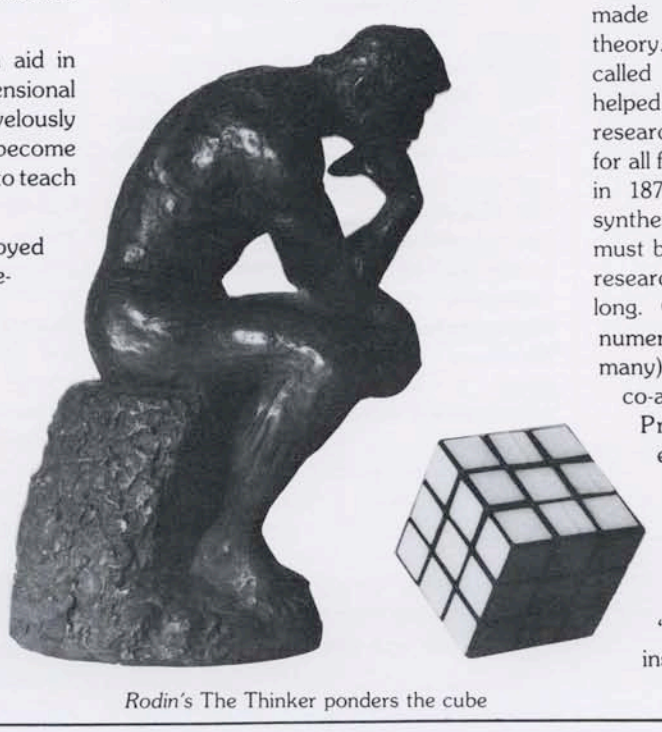


Photo by Robert Ward

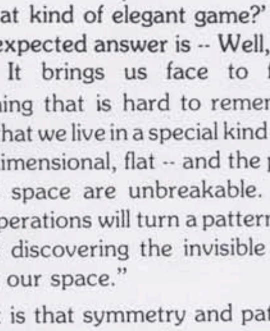
Rodin's The Thinker ponders the cube

table for more moves, brings it out a third time for a last look and then manipulates it for the last time under the table, finally achieving cubical perfection.

Is this game playing spirit, native to all of us, at the heart of mathematics? Is mathematics a sort of game, albeit with serious applications? I think that it is.

I am reminded of Jacob Bronowski who considers this question in his beautiful work, so optimistic for mankind, *The Ascent of Man*. At one point Bronowski is explaining symmetry in nature and art. He takes us to the Alhambra, where in the baths of the harem we see motifs of "wind-swept" triangles in perfect hexagonal collaboration filling the walls. He points out the color pattern of the triangles and the three-fold rotational symmetry it displays. Here in the simple geometric designs the Arab artist and mathematician are fused together. In this way they interpreted the symmetry of space. And then to quote Bronowski, "At this point the non-mathematician is entitled to ask, 'So what? Is that what mathematics is all about? Did Arab professors, do modern mathematicians, spend their time with that kind of elegant game?' To which the unexpected answer is -- Well, it is not a game. It brings us face to face with something that is hard to remember, and that is that we live in a special kind of space -- three-dimensional, flat -- and the properties of that space are unbreakable. In asking what operations will turn a pattern into itself we are discovering the invisible laws that govern our space."

So it is that symmetry and patterning in the real world and in art have a mathematical expression and this mathematics, group theory, not only serves to describe the objects but also to reveal the very nature of the thing and to point out what is and is not possible in creation.



RESEARCH NOTE

A New Perspective on Revenge and Justice in Homer

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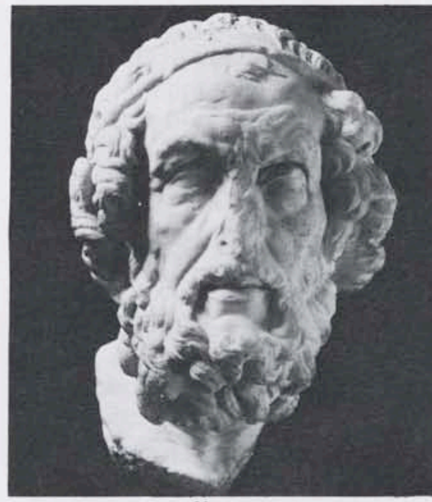
Most of us are aware that our idea of justice comes largely from Ancient Greece. But we might be surprised at how old Greek justice really is. Classical Athens (490-323 B.C.), to which we owe much of our understanding of justice, was itself heir to a system of revenge justice that was older still -- perhaps as old as the Mycenaean period (1200-1100 B.C.). The record of this period is sparse, and with the exception of a few graves and ruined palaces, all that we know of Mycenaean life is found in the oral poetry of Homer.

Because the Mycenaeans were illiterate, the tales of the warrior kings preserved in Homer served as storage mechanisms for social values. Much of the behavior which these tales idealized was aggressive and retaliatory. Both the *Iliad* and the *Odyssey* depict heroes who seek enormous and violent revenge on their enemies. Achilles, for example, kills Hektor to avenge the death of Patroklos and then mutilates his body by dragging it unmercifully around the walls of Troy. Odysseus returns to Ithaca after a twenty year absence and not only kills all one hundred and eight of Penelope's noble suitors but then slaughters and mutilates his own disloyal servants.

To modern readers the severity of revenge, the sensitivity to insult, and the overweening concern for honor with which these heroes are preoccupied seem extreme, but a study of heroic behavior shows that, while not yet the equivalent of a justice in the modern sense, revenge was part of a developing concept of retributive justice based on fairness and reciprocity. To heroes such as Achilles, Agamemnon, and Odysseus, revenge was not only an expression of personal anger but a matter of necessary reciprocity and punishment taken in behalf of the group and accomplished according to certain rules. This is not to say that in such a primitive period social proprieties were always observed, or even consistent, or that the Homeric hero's understanding of his motivations was clear, but a careful examination of the explanations which the heroes give for their actions does indicate that revenge was a serious moral matter.

Revenge was a means of reciprocal justice dependent on fair measure and at its best it



Courtesy, Museum of Fine Arts, Boston. H. L. Pierce Fund.

Homer

punished the aggressor, restored honor, and maintained social balance. Its importance as a moral issue in Homeric society is indicated by attempts to control its extremes in order to guarantee fairness. The accounts in Homer show that revenge was accompanied by proprieties meant to guarantee that it be neither too lenient nor too severe. Sometimes the Homeric hero was tempted by more immediate gratifications such as a large ransom to forget the degree of punishment that responsible revenge demanded. Twice in the *Iliad* Agamemnon is forced to argue for proper severity. In the first case Menelaos, the aggrieved party in whose cause the Trojan War is being fought, is about to accept a ransom and thus spare a captive Trojan. Agamemnon reminds his brother that the only proper revenge is death for all Trojans. Menelaos agrees, kills his captive, and gives up the profit that the ransom would have brought. In the second case Agamemnon himself kills two Trojan boys and foregoes the ransom of gold and wrought iron which they have offered. Although the separation between just reprisal and material profit is not always clear, both these instances in the *Iliad* seem meant to demonstrate that proper revenge was not a matter of personal gain and that it required some sacrifice from the avenger as well as from his victim.

As there were limits on clemency, there were also safeguards against undue cruelty. Passages in both the *Iliad* and the *Odyssey* indicate a concern that punishment not