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An Exploration of Manipulatives in Math Education

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Abstract

Pre-existing literature has shown that the education system needs to re-evaluate mathematical teaching practices in a manner that can boost students’ confidence in mathematics. This research investigates the use of manipulatives in reducing students’ anxiety by increasing their learning experience and engagement in mathematics. The purpose of this article is to explain the interconnectedness of math manipulatives, student engagement, and problem-solving. When manipulatives, student engagement, and problem-solving are in harmony, students can achieve intellectual comprehension of abstract mathematical concepts. The article concludes with a list of recommendations to strengthen mathematical teaching practices.

Key words: manipulative, student engagement, problem-solving, math, teacher, student

1. Introduction

“I hear and I forget. I see and I remember. I do and I understand.” -Confucius (551-479 BC)

Many students find themselves struggling to understand abstract mathematical concepts. To fully grasp a concept, students must go beyond hearing and seeing mathematics, and actively participate in mathematics problems. One way to foster students’ understanding is to present problems in a way that students can comprehend easily. Manipulatives are tools that can demonstrate how effective this aid can be in the learning process. With some help and support, teachers can adjust their mathematical teaching so they can engage students with the use of manipulatives, and in return create stronger and more confident problem solvers.

This paper is organized into seven main sections: introduction, key definitions, manipulatives, student engagement, problem-solving, conclusion, and references. Each of the main areas (manipulatives, student engagement, problem-solving) is accompanied by sub-sections to highlight what the literature has indicated. The benefits and principles of effectiveness for manipulatives, as well as types of manipulatives are explored in the manipulatives section. Next, in the student engagement section, concrete, representational, abstract (CRA) instruction is examined. In the problem-solving section, the followings are discussed: challenging tasks, persistence, the teacher’s role in fostering persistence, and Polya’s method. In the conclusion section, some aspects concerning manipulative uses in mathematics are presented for teachers to consider.

2. Key Definition

One prominent issue teachers face today is keeping students engaged during mathematics lessons. Teachers need the right tools to help their students remain engaged. This can be particularly difficult with mathematics since concepts are so abstract, but research has shown that one specific tool called a manipulative is effective. A manipulative is a physical object that is used to engage students in the hands-on learning of mathematics (Jones & Tiller, 2017). The main purpose of a manipulative is to introduce, practice, and/or remediate a mathematical concept.
concept the manipulative is meant to assist with, such as time and equations (Benders & Craft, 2016; Satsangi et al., 2016).

The second benefit is that manipulatives reduce mathematics-related anxiety (Boggan et al., 2010). Mathematics anxiety varies from person to person, but is described as a feeling of disorientation, in which panic sets in, and students no longer engage with learning. Mathematics anxiety has a negative impact on the students who experience it because they tend to avoid and dislike math (Barrett, 2013). Academic literature suggests that using manipulatives is a constructive approach to help students overcome misunderstood concepts and anxiety (Finlayson, 2014; Blazer, 2011). As a result of math anxiety being reduced, the classroom environment is happier.

The third benefit is that manipulatives provide the “value of interaction” when used over a long period of time. Manipulatives enable students to interact with mathematics on a deeper level, which produces opportunities for hands-on learning, discussion, and collaboration. Research shows that using manipulatives on a long-term basis is more beneficial than using them on a short-term basis. Moore (2014) states “When students are exposed to hands-on learning on a weekly rather than a monthly basis, they prove to be 72% of a grade level ahead in mathematics” (Moore, 2014, p. 2). For math manipulatives to be effective, it is essential that they are frequently used in classroom instruction and practice (Moore, 2014).

The fourth benefit is that manipulatives enhance learning. Learning is enhanced by increasing
retention, content knowledge, and critical thinking. Studies have shown that there are gains in both long-term and short-term student retention of material when using manipulatives (Boggan et al., 2010). When students are utilizing manipulatives, they tend to retain more knowledge because they are constructing their own thoughts and practicing the same concepts (Kamina & Iyer, 2009). Students typically understand the content being taught better when they fully retain it. If a student has a good grasp of the material, they tend to show improved retention in content knowledge (Kilgo & White, 2014). Critical thinking increases when manipulatives are used in a classroom because the relation between a manipulative and a concept may not be obvious (Kilgo & White, 2014). It may require some out of the box thinking if the relationship is not immediately understood. These benefits indicate that manipulatives are incredible assets to students and indicate that manipulatives support students’ academic growth.

3.2 The Three Principles for Effectiveness

Next, the three principles for maximizing the effectiveness of manipulatives are discussed in this section. The first principle is to use a manipulative consistently, over a long period of time. Using the same or similar manipulative leads to a deeper understanding of the relation between the manipulative and the mathematical concept because they have multiple opportunities to compare them (Laski, et al., 2015). Progression happens over time, and students need that time to make connections between a math manipulative and a math concept.

The second principle is to begin with highly transparent, concrete representations and move to more abstract representations over time while avoiding manipulatives that resemble everyday objects. Although concrete representations of mathematics are important for helping students, research suggests that instruction should progress to abstract representations over time (Laski, et al., 2015). In other words, instruction begins with concrete manipulatives, and students gradually move to more abstract symbols (Willingham, 2017). Educators may think that perceptually rich manipulatives are always the way to go, meaning they are covered with lots of details. However, recent research indicates that manipulatives that represent everyday objects may impede learning (Laski, et al., 2015; Willingham, 2017). For example, a study found that children who solved word problems involving money using real dollar bills and coins made more mistakes than those who solved the same problems using more basic representations such as white pieces of paper with only numbers on them (Laski, et al., 2015). The reason for this may be because real dollar bills and coins have distinctive faces, colors, and textures whereas white pieces of paper all look and feel the same. Because of this, research suggests that manipulatives should be as basic as possible (Laski, et al., 2015).

The third principle is to explicitly explain the relation between the manipulative and the math concept. It is unreasonable to expect young children to make the relation between the concrete material and the mathematical concept it represents without explicit guidance (Laski et al., 2015). Explicit statements about how the manipulative represents the
mathematical concept helps direct children’s attention to the relevant features of the manipulative. This promotes learning by allowing children to cognitively focus on the mathematics rather than try to figure out what the relation is. Teachers can use these principles and research to guide their decisions when it comes to using math manipulatives in the classroom, and adjust their practices as necessary.

3.3 Types of Manipulatives

There are a vast variety of manipulatives that can be used during mathematics lessons. Teachers need to know what types of manipulatives are available so they can access them for their own classrooms. Manipulatives should foster children’s conceptual understanding in specific areas such as numbers and operations, patterns, geometry, measurement, probability, reasoning, and more (Boggan, et al., 2010). There are certain manipulatives used in each of these areas. Base 10 blocks (seen in Figure 1), two-color counters (seen in Figure 2), and pattern blocks (seen in Figure 3) are common manipulatives seen in elementary classrooms. These manipulatives, photographed by the author in 2021, can be seen below. Working with manipulatives provides a strong foundation for students to master specific concepts in mathematics and achieve success.
4. Student Engagement

4.1 Concrete, Representational, Abstract (CRA) Instruction

One of the effective teaching methods that can engage students is the concrete, representational, abstract (CRA) instruction, which was first proposed by educational theorist Jerome Bruner. Jerome Bruner (1915-2016) formed the foundation for CRA instruction with his three modes of representations: enactive, iconic, and symbolic (Gibbs, 2014; Mcleod, 2019). CRA instruction, also known as concreteness fading, is a process for teaching and learning mathematical concepts. It moves students away from relying on manipulatives to representing mathematical concepts using abstract symbols (Kim, 2020). Concepts are developed through a progression of three distinct stages: concrete, representational, and abstract (Hurrell, 2018).

The first stage of CRA instruction is the concrete stage and starts with manipulation of concrete materials. The concrete stage has been the theoretical basis for the use of manipulatives in learning mathematics (Leong et al., 2015). This is where manipulatives are used in purposeful activities through senses of sight, touch and/or sound (Hand2mind, n.d.; Hurrell, 2018). Being introduced to and working with manipulatives are critical first steps to develop students’ understanding of mathematical concepts. Manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning (Boggan et al., 2010).

The second stage of CRA instruction is the representational stage. Students transition to this stage after they have shown conceptual understanding towards the mathematical task using concrete manipulatives. This moves students to using pictures such as tallies, dots, and stamps to replace the manipulatives used in the previous stage (Jones & Tiller, 2017). Other visual representations that can be used in this stage are images, graphs, diagrams, and tables (Cabahug, 2012). In the representational stage, students should be comfortable with solving problems using visual representations. Students should also be able to demonstrate how they can both visualize and communicate the concept at a pictorial level (Hand2mind, n.d.). The purpose of this stage is to gradually move students away from relying on manipulatives and building up their skillset to reach the final stage of the process.

The third, and final stage of CRA instruction is the abstract stage. Students transition to this stage by using abstract symbols with their drawing to explain their reasoning. Students at the abstract level no longer need pictures or manipulatives to solve the problem (Jones & Tiller, 2017). This transition is the most challenging aspect of the CRA sequence because students are required to generalize their understanding in succinct ways (Hurrell, 2018). Instead of manipulatives or pictures, mathematical symbols (numerals, operation signs, etc.) are adopted and used to express the concept. In this stage, students demonstrate their understanding by using the language of mathematics (Hand2mind, n.d.).
The three stages of the CRA process are concrete, representational, and abstract. A summary of each stage can be seen in Figure 4 below. An example of CRA instruction being used with two-color counters is shown in sequential order with each stage as well. The concrete stage utilizes manipulatives. The representational stage uses visual representations instead of manipulatives. Lastly, the abstract stage uses mathematical symbols such as numerals and operation signs. By following the sequence of CRA instruction, teachers assist students on their progression to each stage, and ensure they achieve the goal of demonstrating understanding at the abstract level.

<table>
<thead>
<tr>
<th>Stages</th>
<th>Key Elements</th>
<th>Simple Problem</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>Chips, Unifix cubes, base ten blocks</td>
<td><img src="image" alt="Concrete Example" /></td>
<td>Here, a ten frame with colored counters is used to show the equation 7 + 3 = 10.</td>
</tr>
<tr>
<td>Transition to</td>
<td>Use of concrete and</td>
<td><img src="image" alt="Transition Example" /></td>
<td>Once the concrete materials have been used, students begin to draw their own ten frames using the concrete model as a guide.</td>
</tr>
<tr>
<td>Representational</td>
<td>representational materials together</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Representational</td>
<td>Tallies, dots, circles, stamps</td>
<td><img src="image" alt="Representational Example" /></td>
<td>At the representational level of CRA, the student is comfortable using pictures to solve the problem.</td>
</tr>
<tr>
<td>Transition to</td>
<td>Use of representational and abstract</td>
<td><img src="image" alt="Transition Example" /></td>
<td>Students now start using abstract symbols (numbers in standard form) with their drawings to explain their reasoning.</td>
</tr>
<tr>
<td>Abstract</td>
<td>mathematical symbols</td>
<td><img src="image" alt="Abstract Example" /></td>
<td>Students at the abstract level of CRA no longer need pictures or manipulatives to solve the problem.</td>
</tr>
</tbody>
</table>

*Figure 4: CRA Instruction Stages. Jones & Tiller, 2018, p. 19.*

5. Problem-Solving

5.1 Challenging Tasks

- Students often need to problem-solve because they need to overcome a challenging task. A challenge teachers face is introducing challenging tasks in a way that makes them accessible, rather than daunting (Cheeseman et al., 2016). Sullivan et al. (2011) identified the requirements of a challenging mathematical task. They require students to:
  - plan their approach; especially, sequence more than one step;
  - process multiple pieces of information, with an expectation that they make connections and see mathematical concepts in new ways;
applying themselves, believing that they can succeed, and putting in effort to learn (Roche & Clarke, 2014). Enduring uncertainty and overcoming obstacles are recognized as key practices that support learning. Students make meaning through productive struggle as they engage with mathematical ideas that are within reach, but not yet well formed (DiNapoli, 2019). Since math requires an overt amount of persistence for every student, manipulatives are tools to not shy away from when it gets hard. The amount of persistence varies by degree and by topic area for each student, but manipulatives open a door for accessibility and help students persist through struggle.

5.3 The Teacher’s Role in Fostering Persistence

Teachers play a significant role in the problem-solving process by encouraging students to persist. The National Council of Teachers of Mathematics (NCTM) recommends that teachers need to provide opportunities for productive struggle. Productive struggle is essential to learning mathematics with understanding (Livy et al., 2018). Providing consistent opportunities for students to engage with unfamiliar mathematical tasks encourages problem-solving strategies. After providing challenging tasks, teachers can take a step back and allow for students to go through productive struggle. As one student wrote in a written reflection: “We do learn more when we’re confused and we’ve got to work our way out of it” (Roche & Clarke, 2014, p. 7). As this student has stated, students do learn when they are confused even if it does not seem like it. Being confused is productive in mathematics, and means students need to critically think about the challenging task.

• choose their own strategies, goals, and level of accessing the tasks;
• spend time on the task and record their thinking;
• explain their strategies and justify their thinking to the teacher and other students;
• extend their knowledge and thinking in new ways (Sullivan et al., 2011, p. 34).

Teachers can take these requirements and foster a classroom environment that promotes productive struggle by providing students with challenging tasks. Providing challenging tasks is sometimes disconcerting for teachers because they see students struggling and want to intervene too soon. However, intervening too soon is detrimental to the learning taking place in the discomfort. Productive struggle enables students to work through challenging problems that they have not seen before.

5.2 Persistence

To effectively overcome productive struggle, students must be able to persist through the problem-solving process. Persistence, also known as perseverance, is productive struggle in the moment while facing mathematical obstacles, setbacks, or discouragements. In other words, it is the ability to “stick with it” to reach a solution (DiNapoli, 2019). One major influencer on the topic of perseverance is Angela Duckworth. Her famous book entitled Grit: The Power of Passion and Perseverance highlights exactly what perseverance is and why it matters (Duckworth, 2016). Persistence is also described as a category of student actions that include concentrating,
It is vital for teachers to allow time for confusion without giving away the answer too quickly. To ensure a strong mathematical focus within the classroom, students need to think for themselves. For this reason, teachers need to avoid the kind of relationships that encourage dependency. Students may be highly engaged in a problem when they seek help, but they may also be over-dependent on the teacher. Therefore, teachers are encouraged to allow students to enter a zone of confusion. Ingram et al. (2016) defines a zone of confusion as “A state of confusion before a pathway for solving the problem has been identified” (Ingram et al., 2016). The zone of confusion is key because an important part of maintaining the challenge of a task is for students to make decisions about how to approach the problem. In return, students learn to appreciate and evaluate different thoughts when they undergo decision making since the mathematical reasoning is in their hands (Cheeseman et al., 2016).

It is important for students to persist until they are certain that they have fully solved the problem and feel confident with their answer. Students can develop persistence as they work and discover multiple answers that satisfy the problem. Furthermore, when students acknowledge that there are different problem-solving strategies, they have more choices to work with. Some students are better at verbalizing their strategy and thinking processes, but others may use visual aids such as tables or ordered lists to demonstrate their reasoning (Suh et al., 2011). Overall, students are encouraged to approach a problem believing that they can succeed and recognize that learning mathematics takes effort.

5.4 Polya’s Method

It is critical for students to persist through problem-solving, and for teachers to foster students’ persistence. Once students are met with a challenging task, they need to have an effective method to work through it. One strategy is Polya’s method, named after George Polya. Manipulatives and Polya’s method can be used simultaneously to conquer a problem. (In’am, 2014). Polya’s method is a four-step process for teaching and assessing problem-solving in mathematics. The four steps are: understanding the problem, devising a plan, carrying out the plan, and looking back (Ortiz, 2016). This method guides students in solving problems, and complete the result by looking back (In’am, 2014).

The first step of Polya’s method is to understand the problem. This is the step where students engage with the problem before attempting to solve it (Ortiz, 2016). By looking for information, students take a step forward to understand the problem to be solved (In’am, 2014). Tohir et al. (2020) lists some questions to guide students in understanding the problem:

- What is the unknown?
- What are the data?
- What is the condition?
- Is the condition sufficient, insufficient, redundant, or contradictory to determine the unknown? (Tohir et al., 2020, p. 1736).

The second step of Polya’s method is to devise a plan. After identifying the problem, the next step is
to plan appropriate strategies to solve the problem. Two strategies students may use are making an appropriate diagram and making an analogy with similar problems. It is important to note that different problems require different approaches, and not one strategy will be used to solve all problems (In’am, 2014). The student should try to find connections between the data and the unknown (Ortiz, 2016). Tohir et al. (2020) and Ortiz (2016) lists some questions to guide students in devising a plan:

- Have I seen the same, similar, or related problem before?
- Do I know a theorem that could be useful? (Tohir et al. 2020, p. 1736).
- Could I restate the problem?
- Do I know the vocabulary in the problem? (Ortiz, 2016, p. 6).

The third step of Polya’s method is to carry out the plan. Understanding the problem, and then planning to solve it is not useful if the plan is not implemented. Therefore, the next step is to implement the strategy from the prior step to solve the problem (In’am, 2014). Students should check each step of the solution plan to make sure they clearly see each step is correct (Ortiz, 2016). Tohir et al. (2020) lists the following guiding questions for carrying out the plan:

- Can I see that the step is correct?
- Can I prove that it is correct? (Tohir et al., 2020, p. 1736).

The fourth and final step of Polya’s method is reflection. It is critical for students to examine and review the solution they obtain at the end of problem-solving. This step might be done by using the answer through inverse method. By using the inverse method, students can check whether the answer is appropriate compared to the expected solution. For example, a student may look back on a multiplication problem by using division (In’am, 2014). The following guiding questions are from Tohir et al. (2020) for looking back:

- Can I check the result/argument?
- Can I derive the result differently?
- Can I use the result, or the method, for some other problem? (Tohir et al., 2020, p. 1736).

A challenging task will often stump students to the point where they no longer want to engage in problem-solving. Polya’s method provides students a set of steps to follow, so they can obtain a solution to a challenging task. Understanding the problem, devising a plan, carrying out the plan, and looking back help students think like mathematicians. Students must ask themselves if they truly feel comfortable at any given step before moving on to the next one.

6. Conclusion

The literature has indicated that the use of manipulatives in the classroom engages students to persist in the problem-solving process during mathematics. A manipulative is a valuable tool that enhances mathematical learning. First, manipulatives are often used in classrooms because they have multiple benefits that enhance a student’s learning experience. Those benefits include: (1) increasing math achievement, (2) reducing math anxiety, (3)
alterations. Based off of the literature, current and future teachers can consider:

- Re-evaluating their mathematical teaching practices.
- Incorporating manipulatives into their classrooms.
- Using CRA instruction to engage students in mathematics.
- Using Polya’s method to engage students in problem-solving.

By considering the recommendations listed above, teachers can help improve students’ attitudes towards mathematics and opportunities for success.

After completing a full literature review, more work must be done to help students reach their full potential. One approach that can be considered is to perform studies in classrooms to see if the literature accurately reflects the effectiveness of manipulatives. Studies can be done by observing and surveying teachers at all grade levels. Another approach is to create professional development workshops on manipulative use in the classroom to help teachers build up their skillset, confidence, and comfortableness while teaching with manipulatives. There are endless learning opportunities in mathematics when manipulatives and constructive instructional methodologies are used jointly.
7. References


Figures:

*Figure 1*: Base 10 Blocks.

*Monte, J. (2021).* *Base 10 Blocks.* [Photo].

*Figure 2*: Two-Color Counters.

*Monte, J. (2021).* *Two-Color Counters.* [Photo].

*Figure 3*: Pattern Blocks.

*Monte, J. (2021).* *Pattern Blocks.* [Photo].

*Figure 4*: CRA Instruction Stages.

About the Author

Jade Monte is double majoring in Elementary Education and Mathematics. Her two-semester thesis was completed in the 2020-2021 academic year under the mentorship of Dr. Jacquelynne Boivin (Elementary & Early Childhood Education) and Dr. Nguyen Ho (Mathematics). Jade plans to pursue a master’s degree in Education after graduation and then teach at a school on Cape Cod.