Roller Coaster Acceleration

Olivia Briggs

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Roller Coaster Acceleration

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Roller Coaster Acceleration

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1 Introduction

In classical mechanics, Newton’s Second Law (NSL) is often used when talking about acceleration. NSL states that the acceleration of an object is directly related to the net force and inversely related to its mass. It can be written as

\[ \vec{F} = m\vec{a}. \]

This concept can be applied to many real life situations, one of them being roller coasters. Passengers on roller coasters can accelerate in all directions. Some accelerations are due to gravity while others are due to launch systems that apply a mechanical force to a train. Roller coasters are often described by how many G forces riders experience while riding them. A G force is not an actual force but is a measurement of force per unit mass. One G force is equal to the acceleration due to gravity, \( 9.8 \frac{m}{s^2} \), and causes the perception we know as weight. So a coaster that advertises as experiencing 5 G forces will make the rider feel five times heavier than their normal weight. G forces can be felt in all six directions of the \( xyz \) coordinate system.

1.1 History of Coasters

In 1610, an enterprising showman constructed a wooden slide over a wood frame to form a steep 70ft incline. He then poured water over the structure in the winter to coat it with ice and sent the sledders down [Valenti, 1995]. This ride was so popular that he invented the first wheeled cart so it could be ridden in the summer. These structures were nicknamed Russian Mountains. The term “roller coaster” didn’t emerge until 1884 when Philo M. Stevens built a circular railway in Chicago and patented it to the Roller Coaster of America Company. The first modern day roller coaster was opened at Coney Island in Brooklyn, New York in 1884. The first looping roller coaster was introduced in 1895 by the name of “Flip-Flap”. It resided at Sea Lion Park in Coney Island, New York. Flip-Flap consisted of a lift hill, a single drop, and a 25 meter diameter circular loop. It can be seen in Figure 1.

Figure 1: The first looping coaster “Flip Flap”. Adapted from: https://en.wikipedia.org/wiki/Flip_Flap_Railway

This ride was causing severe neck and back pain, so only one cart that sat two people was allowed through at once. Despite the unpleasant experience, replicas of this ride were still being made, and it wasn’t until 1901 when the first elliptical loop was introduced on a coaster. This coaster was named “Loop the Loop” and was also located in Coney Island.
The elliptical loop reduced turbulence and extreme G forces that riders experienced on circular loops. Although this ride was still uncomfortable, it introduced a revolutionary concept that has been modified over time. Loop the Loop can be seen in Figure 2.

Figure 2: The first coaster with an elliptical loop “Loop the Loop”. Adapted from: https://www.heartofconeyisland.com/loop-the-loop-roller-coaster.html

The very first roller coaster loops laid out the groundwork for more complex elements. The classic elliptical loop, with some adjustments, is the big idea behind all modern day vertical loops. Since then, coasters have been evolving both technologically and structurally. In 1977 the first launching coaster was installed by Arrow Dynamics at Six Flags New England by the name Black Widow. The launch was designed as a means to get rid of the lift hill. It was also useful if engineers were tight on space. Some would say launches provided a more thrilling experience than the traditional lift hill. From 1977 to 1997, multiple different launch systems were invented including the hydraulic, pneumatic, flywheel, linear induction motor (LIM), and linear synchronous motor (LSM) models. In June of 2019, the most expensive coaster at $300 million dollars opened to the public. It featured a record of 7 LSM launches on one coaster. This coaster is located in Universal’s Islands of Adventure in Orlando, Florida and can be seen in Figure 3.

Figure 3: “Hagrid’s Magical Creatures Motorbike Adventure” in Orlando, Florida. Adapted from: https://www.reddit.com/r/rollercoasters/comments/bzk9nn/dont_forget_hagrids_magical_creatures_motorbike/
1.2 Flow of Paper

The purpose of this thesis is to explore the physical applications of acceleration on roller coasters. Section 2.1 explores the application of acceleration due to gravity. To do this, a simple model of a roller coaster hill and loop is introduced. Using conservation of energy, equations are derived that give how many G forces a rider experiences while traveling in a loop. This section also introduces the first ever looping coaster and explains the reasons why it was so unsafe and had to be removed.

Section 2.2 shows a brief introduction of how passengers sitting on different parts of the coaster train can experience different G forces at the same time. Some equations are shown for these forces.

Section 2.3 introduces alternate loops shapes other than circular ones that are much safer for passengers. Differential equations are given that can be used to model these different loop shapes. The two types of loops modeled are the constant G force loop, and the constant centripetal acceleration loop. They are modeled using the Runge Kutta method with Python. G force graphs are made in excel to show what riders actually feel on each loop.

Section 3.1 introduces the linear induction motor, which is a widely used method of launching coaster trains. This section talks about the history of how they came to be. This section highlights important people who contributed to this technology and also lists some of the LIMs applications.

Section 3.2 explains the inner workings of an LIM and the underlying physics of how it functions. This section also explains different types of LIMs.

Section 3.3 describes the limitations and efficiency flaws that LIMs face. End effect, airgap length, and slip are explored.

Finally, section 3.4 models some parameters of a short secondary double sided LIM. Parameters are given and the equation for thrust output is derived. Two graphs of different synchronous speeds are compared and two graphs of different airgap lengths are compared.

All codes and calculations can be found in the Appendix.
2 Acceleration Due to Gravity

2.1 Circular Loops

When a roller coaster cart travels in a loop, riders experience a centripetal acceleration that pushes them into their seats. In introductory physics classes, a perfectly circular loop is often used in order to make calculations of centripetal acceleration easy.

Using conservation of energy, assuming the velocity at the top is equal to 0 m/s, and ignoring friction and air resistance, the kinetic energy of the cart at any height on the hill, $h$, is equal to

$$E_k = mgh_0 - mgh = \frac{1}{2}mv^2.$$ 

This means that the velocity of the cart at any point is

$$v = \sqrt{2g(h_0 - h)}. \quad (1)$$

When the cart enters the loop, $h = 0m$, so the velocity at loop entrance is equal to

$$v_0 = \sqrt{2gh_0}. \quad (2)$$

The cart experiences centripetal acceleration while traversing the loop. Drawing a simple free body diagram as seen in Figure 4, Newton’s Law gives us

$$\Sigma F = ma_c,$$

$$F_N - mg\cos(\theta) = \frac{mv^2}{r}.$$ 

The normal force, or the force felt by the rider is equal to

$$F_N = \frac{mv^2}{r} + mg\cos(\theta)$$

The number of G’s felt by the rider is the normal force divided by $mg$,

$$\#\ of\ G’s = \frac{F_N}{mg} = \frac{v^2}{rg} + \cos(\theta) \quad (3)$$

Replacing $v^2$ with $2g(h_0 - h)$ from equation (1) we get

$$\#\ of\ G’s = \frac{2(h_0 - h)}{r} + \cos(\theta).$$
Next, looking at a circular loop, the height off the ground, $h$, is equal to $r - r \cos(\theta) = r(1 - \cos(\theta))$. Taking equation (2), it can be derived that $h_0 = \frac{v_0^2}{rg}$. Using these two equations as substitutions, the number of G forces felt by a rider is equal to

$$\# \ of \ G's = \frac{v_0^2}{rg} + 3 \cos(\theta) - 2. \ (4)$$

Using equation (4), the G forces felt by a rider on the loop of Flip Flap Railway can be found. According to the Roller Coaster Database it has a radius of 12.5 meters and is estimated to have travelled at a velocity of $40 \frac{m}{s}$ into the loop. Figure 5 shows the G forces felt on Flip Flap as a function of radians. This was graphed using Python in Jupyter Notebook.

This graph reveals many important things about circular loops. First, the largest positive G force on the loop is around 14 G’s. Depending on the individual’s ‘g-tolerance’, the oxygen supply to the head may cease completely at 5–6g’s, resulting in unconsciousness if extended in time [Pendrill, 2005]. Another issue with circular loops is that entering the loop from a horizontal track would automatically apply the maximum G force, giving the rider an instantaneous feeling of 14 G’s. This ride was obviously not safe.

### 2.2 G Forces in Different Places on The Train

![Figure 6: A circular loop with radius R, center O, and top T. The train’s center of gravity is shown as G. [Pendrill, 2013]](image)

The G forces that riders experience depend on where they sit in the train. As a train travels through a loop, it slows down as it reaches the top, then speeds up again on
the way down. If a train is long, the front, middle, and back of the train travel across
different spots of the loop at different velocities, as the energy changes from kinetic to
potential. The front of the train will enter the loop at a faster velocity than the rest of
the train and the back of the loop will exit the loop at a faster velocity than the rest of
the train. For simplicity, on a loop with a circular shape, the normal force on a rider in
the middle of the train while traversing the top of a loop is

\[ F_{n(middle)} = m \left( \frac{v_0^2}{R} + 2g(1 - \cos(\theta)) \frac{\sin(\alpha)}{\alpha} \right) \]

whereas the normal force on a rider in the front or back of the train would be

\[ F_{n(front/back)} = m \left( \frac{v_0^2}{R} + 2g(1 - \cos(\theta)) \frac{\sin(\alpha)}{\alpha} + g \cos(\theta + \alpha) \right). \]

The angle \( \theta \) is the angle between the middle of the train and the highest point, and
the angle \( \alpha \) relates the radius of the loop to the length of the train by \( L = 2\alpha R \). This
can be seen in Figure 6. Roller coaster engineers need to make sure that the G forces are
safe for riders in the front, middle, and back of the train. Figure 7 shows the forces that
the front, middle and back of the train feel at different spots.

![Figure 7: Forces felt by riders in the front, middle, and back of the train at different spots. The figure to the left shows when the front of the train is just reaching the top, and the figure to the left shows when the middle of the train is at the top.](image)

### 2.3 Alternate Loop Shapes

Circular loops were extremely uncomfortable and dangerous, which is why roller
coaster engineers now design tear drop shaped loops with more gradual transitions of
radii. The American Society for Testing and Materials, ASTM, now has standards limiting
G force exposure on amusement rides. Loops are designed to accomplish various goals.
Engineers pick whether they want a loop to have a constant G force, constant acceleration,
or a range of G forces, and reverse engineer it. These various curves can be described
through the differential equations,

\[
\begin{align*}
\frac{dx}{ds} &= \cos(\theta), \\
\frac{dy}{ds} &= \sin(\theta), \\
\frac{d\theta}{ds} &= \frac{1}{r}.
\end{align*}
\]

The difference between loop types is reflected in the expression for the curvature (i.e. \( \frac{1}{r} \)).
[Pendrill, 2005].
2.3.1 Loops With a Constant Centripetal Acceleration

For a loop with constant centripetal acceleration, the formula for centripetal acceleration can be used

\[ a_c = \frac{v^2}{r} = Cg \]

where \( C \) is a chosen constant. Solving for \( r \) gives

\[ r = \frac{v^2}{Cg} = \frac{1}{C} \left( \frac{v_0^2}{g} - 2y \right) \]

where \( y \) is the height off the ground. This equation gives the radius as a function of height. The three differential equations become

\[ \frac{dx}{ds} = \cos(\theta), \quad \frac{dy}{ds} = \sin(\theta), \quad \frac{d\theta}{ds} = \frac{1}{r} = \frac{C}{\left( \frac{v_0^2}{g} - 2y \right)} . \]

![Roller Coaster Loop with a Constant Centripetal Acceleration of 2 G's](image1)

Figure 8: Example of a loop with a constant centripetal acceleration of 2 G’s when the train enters the loop at \( 20 \text{ m/s} \). Axes represent meters in 2D.

Figure 8 shows a loop with a constant centripetal acceleration of 2 g’s, and initial velocity of \( 20 \frac{\text{m}}{\text{s}} \) that was graphed using the Runge Kutta 4th Order Method in Python Jupyter Notebook.

![G Forces Felt on A Loop With Constant Centripetal Acceleration](image2)

Figure 9: Graph of the G forces felt by a rider in a loop with a constant centripetal acceleration of 2 G’s.
Using equation (3), replacing $v^2$ with $(v_0^2 - 2gh)$ the G forces felt throughout the loop can be graphed. Figure 9 shows the number of g’s felt versus radians. The G forces are much safer on this loop, with a range of only 2 g’s compared to Flip Flap’s range of 6 g’s as seen in Figure 5.

### 2.3.2 Loops With a Constant G Force

If an engineer wants to design a loop with a constant G force, they will need to manipulate equation (3). Solving for $r$ and using $v^2 = v_0^2 - 2gy$ where $y$ is the height off the ground

$$r = \frac{v_0^2 - 2gy}{(G - \cos(\theta))g}.$$

The three differential equations then become

$$\frac{dx}{ds} = \cos(\theta), \quad \frac{dy}{ds} = \sin(\theta), \quad \frac{d\theta}{ds} = \frac{1}{r} = \frac{(G - \cos(\theta))g}{v_0^2 - 2gy}.$$

![Graph of the G forces felt by a rider in a loop with a constant G force of 3.5 G’s.](image)

Figure 10: Example of a loop with a constant G force of 3.5 G’s when the train enters the loop at 20 m/s. Axes represent meters in 2D.

Figure 10 shows a loop with a constant G force of 3.5 g’s, and initial velocity of $20\frac{m}{s}$ that was graphed using the Runge Kutta 4th Order Method in Python Jupyter Notebook.

![Graph of the G forces felt on a Loop with Constant G Force](image)

Figure 11: Graph of the G forces felt by a rider in a loop with a constant G force of 3.5 G’s.
The G forces felt throughout the loop can be graphed using equation (3), replacing \( v^2 \) with \( (v_0^2 - 2gh) \). Figure 11 shows the numbers of G’s felt versus radians. As expected, the G force felt by riders is constant throughout the loop. This method allows engineers to pick what safe G force they want a loop to exert. Having a constant G force is also much safer than a wide range of G forces.

### 2.3.3 Clothoid Loops

Now that there is a method to design loops with safe G forces, there is still a method needed to eliminate the jump from zero G’s to the maximum G force at the loop’s entrance. Clothoid loops are a used by many roller coaster engineers to fix this issue. They are designed by using the Euler Spiral, which is used in engineering to provide a smooth transition from a horizontal line to a circular curve. They can be mathematically represented by the integrals [Levien, 2008],

\[
\begin{align*}
x &= \int \cos \left( \frac{s^2}{2a^2} \right) ds \\
y &= \int \sin \left( \frac{s^2}{2a^2} \right) ds
\end{align*}
\]

where \( s \) is the arclength and \( a \) represents some known constants.

Figure 12: Euler Spiral. Adapted from: https://en.wikipedia.org/wiki/Euler_spiral

Figure 12 shows a picture of the Euler Spiral. Clothoid loops have a larger radius at the bottom to reduce intense G forces when entering the loop, and a smaller radius at the top to keep the train from slowing down too much. The radius of curvature is inversely proportional to the distance from the center of the spiral. These loops are used in multiple ways. They can either be used to connect tracks of different curvatures to create a smooth transition, or they can be used to design complete loops. When used to design loops, they can be extended throughout the loop, or just used at the beginning and be matched to a circular top.
3 Acceleration Due to Launch Systems

3.1 Linear Induction Motor Background

One of the most popular launch systems used for roller coasters is the linear induction motor (LIM). The history of LIMs can be traced back to the 1840s by the work of Charles Wheatstone at Kings College in London, though his model was inefficient. It wasn’t until 1905 that the US patent 782312 was introduced by Alfred Zehden [Zehden, 1905], and 1935 that the first working model was built by Hermann Kemper. The first full size working model was built in the 1940s by Eric Laithwaite, a Professor at Imperial College in London. Since then, LIMs have been in use for high speed trains, people moving systems, aircraft launch systems, and roller coaster launch systems.

3.2 How Do LIMs Work?

In a traditional AC electric motor, electromagnets are positioned around the edge of the motor. This section is known as the stator. The electromagnets are used to generate a rotating magnetic field in the center. The moving magnetic field induces an electrical current in the rotor section, causing it to spin. Linear induction motors took the technology of rotating electric motors and essentially sliced it open [Liasi, 2015]. Figure 13 illustrates this idea.

![Figure 13: Comparison of a traditional rotating electric motor (left) and a linear electric motor (right). Adapted from: https://circuitglobe.com/linear-induction-motor.html](https://circuitglobe.com/linear-induction-motor.html)

The stator is laid out and made up of a flat magnetic core with slots that are straight cut. In these slots are coils with different phases that overlap. This is known as the “primary” of a linear induction motor. The rotor is usually a sheet of aluminum or copper with an iron backing plate. The coils in the primary will have induced eddy currents which in turn create an opposing magnetic field explained by Lenz’s law. Electric current is induced into the aluminum conductors of the secondary due to the magnetic flux. This current interacts with the traveling magnetic wave to create a linear thrust force $F$. If the primary is fixed and the secondary is free to move, the secondary will be moved in the direction of the force. Figure 14 shows an LIM on the track of an inverted coaster.
LIMs can also be double sided (DSLIM) where there are two primaries that the secondary glides through. These are optimal for high speed linear motor applications where no levitation is required. They generate a field on both sides of the secondary resulting in more flux. DSLIMs are often the choice for roller coaster accelerations. Furthermore, there are two options for DSLIMs. There is either a short primary or short secondary DSLIM. In a short primary DSLIM, the secondary extends beyond the field producing part of the primary. In a short secondary DSLIM, the primary extends beyond the secondary.

3.3 Downsides to LIMs

Unlike a circular induction motors, linear induction motors have an “end” to them. This causes something called the end effect. The end effect causes discontinuities in the magnetic field producing part of the LIM or the conducting part of the LIM in a short primary or short secondary LIM respectively [Hamzehbahmani, 2011]. Another challenge LIMs face is optimizing the airgap length between the primary and secondary. A smaller airgap decreases the amount of magnetic flux lost, but increases the chance of the primary and secondary interacting which can cause significant damage [Cao, 2019].

The synchronous speed of an LIM is the speed of the magnetic field in the stator and it depends on the input power frequency and the number of electromagnetic poles in the motor. However, the synchronous speed of the magnetic field in the stator is not the speed of the rotor. If the speeds were equal, there would be no relative motion between the stationary stator and the rotor. This difference in speed between the stator and the rotor is known as the slip. The slip is determined by the load of the train. A stationary train will have a slip of 1 and in all other running conditions the slip will be between zero and 1. The value of the slip affects the thrust, or force applied to the secondary.

3.4 Modeling

A representation of a short secondary DSLIM is shown in figure 15. This is a one dimensional model assuming that the surface current on the primaries is a perfect
sinusoidal distribution driven by a perfect sinusoidal source. The surface current on the primaries and secondary point straight out of the page in the positive z-direction, the secondary moves in the positive x-direction, and the magnetic field is in the negative y-direction. This philosophy is explained by the nature of electromagnetic waves. The electric and magnetic field waves are perpendicular to each other and the direction that the electromagnetic wave travels.

Figure 15: One Dimensional Model of a LIM

Some parameter definitions needed for this derivation are as follows:

- \( g \) = airgap length
- \( d \) = secondary thickness
- \( V \) = shuttle speed
- \( \sigma_s \) = surface conductivity
- \( \tau \) = pole pitch
- \( V_s \) = synchronous speed
- \( k = \frac{\omega}{\tau} \)
- \( \omega = \frac{\pi}{\tau} V_s \)

Maxwell’s equations are a set of differential equations that form the foundation for electromagnetism. They can be used to calculate the electromagnetic fields in an LIM and these equations can then be used to study and model different parameters. The goal of the following derivation is to find the magnetic field of a DSLIM in order to calculate the thrust, or power output. Once the thrust equation is known, the slip and airgap parameters can be investigated. Using the differential form of Ampere’s Law, we obtain equation (5) by taking the curl of the H-field

\[
g \frac{dH_y}{dx} = K_S + K_R
\]

where \( K_s \) is the primary surface current, \( K_R \) is the rotor (or secondary) surface current, and \( g \) is the length of the airgap between the two primary parts. Taking the x-derivative of equation (5) we get

\[
g \frac{d^2H_y}{dx^2} = \frac{dK_S}{dx} + \frac{dK_R}{dx}
\]

The rotor surface current density is proportional to the electric field by

\[K_R = \sigma_s E_R\]
where $\sigma_s$ is the surface conductivity.
Replacing $\vec{K}_R$ in equation (6) with this relationship we get

$$g \frac{d^2 \vec{H}_y}{dx^2} = \frac{d\vec{K}_S}{dx} + \sigma_s \frac{d\vec{E}_R}{dx}. \quad (7)$$

Next, Maxwell’s equations give

$$\nabla \times \vec{E} = \frac{-d\vec{B}}{dt} = -\mu_0 \frac{d\vec{H}}{dt}.$$ 

Since the magnetic field is pointing in the negative y direction we can say

$$\frac{d\vec{E}_R}{dx} = \mu_0 \frac{d\vec{H}_y}{dt}$$

and update equation (7) to

$$g \frac{d^2 \vec{H}_y}{dx^2} = \frac{d\vec{K}_S}{dx} + \sigma_s \mu_0 \frac{d\vec{H}_y}{dt}. \quad (8)$$

Furthermore, the time derivative of the H field is related to the shuttle speed, $V$, by

$$\frac{d\vec{H}_y}{dt} = \frac{d\vec{H}_y}{dx} + V \frac{d\vec{H}_y}{dx}. \quad (9)$$

Taking equation (9) and plugging it into equation (8) we get

$$g \frac{d^2 \vec{H}_y}{dx^2} = \frac{d\vec{K}_S}{dx} + \sigma_s \mu_0 \left( \frac{d\vec{H}_y}{dx} + V \frac{d\vec{H}_y}{dx} \right). \quad (10)$$

The exponential form of the H field and surface current are written as

$$\vec{H}_y = H_0 e^{i(kx - \omega t + \delta)} \quad \vec{K}_S = K_0 e^{i(kx - \omega t)}.$$ 

The H field equation describes a sinusoidal magnetic field strength whose max amplitude is $H_0$. The surface current equation describes a sinusoidal wave whose max amplitude is $K_0$. In these equations, $k$ is the wave number, which is $2\pi$ times the number of wavelengths that fit into a unit length and is equal to $\frac{2\pi}{\lambda}$, $\omega$ is the angular frequency, and $\delta$ is the phase which specifies where the wave is at $t = 0$. Taking the required derivatives gives

$$\frac{d\vec{H}_y}{dx} = ik H_0 e^{i(kx - \omega t + \delta)}$$

$$\frac{d^2 \vec{H}_y}{dx^2} = -k^2 H_0 e^{i(kx - \omega t + \delta)}$$

$$\frac{d\vec{H}_y}{dt} = -i\omega H_0 e^{i(kx - \omega t + \delta)}$$

$$\frac{d\vec{K}_S}{dx} = ik K_0 e^{i(kx - \omega t)}.$$ 

Plugging these into equation (10) we get

$$-g k^2 H_0 e^{i(kx - \omega t + \delta)} = ik K_0 e^{i(kx - \omega t)} + \mu_0 \sigma_s (-i\omega + ikV) H_0 e^{i(kx - \omega t + \delta)}. \quad (11)$$

3 ACCELERATION DUE TO LAUNCH SYSTEMS
This equation simplifies to

$$-gk^2H_o e^{i\delta} = ikK_o + \mu_0\sigma_s(-i\omega + ikV)H_o e^{i\delta}$$  (12)

by canceling out some exponential terms.

Next, by factoring out the term all the way to the right gives

$$-gk^2H_o e^{i\delta} = ikK_o - \mu_0\sigma_s i\omega H_0 e^{i\delta} + \mu_0\sigma_s ikV H_0 e^{i\delta}.$$  

Using Euler’s relation the exponential terms become sines and cosines.

$$-gk^2H_o(e^{i\delta} + i \sin \delta) = \mu_0\sigma_s i\omega H_0(e^{i\delta} + i \sin \delta) + \mu_0\sigma_s ikV H_0(e^{i\delta} + i \sin \delta)$$

Factoring out the sines and cosines this equation becomes

$$-gk^2H_o \cos \delta - gk^2H_o i \sin \delta = ikK_o - \mu_0\sigma_s i\omega H_0 \cos \delta - \mu_0\sigma_s ikV H_0 \cos \delta + \mu_0\sigma_s kV H_0 i^2 \sin \delta$$  (13)

Next, separating the real and imaginary terms helps to solve for $\delta$ and $H_0$. The equation with just real terms looks like

$$-gk^2H_o \cos \delta = \mu_0\sigma_s \omega H_0 \sin \delta - \mu_0\sigma_s kV H_0 \sin \delta$$

By cancelling out $H_0$ in all terms and factoring $\sin \delta$ out on the right side, the equation becomes

$$-gk^2 \cos \delta = \sin \delta(\mu_0\sigma_s \omega - \mu_0\sigma_s kV)$$

Dividing both sides by $\cos \delta(\mu_0\sigma_s \omega - \mu_0\sigma_s kV)$ results in the tangent of $\delta$ equal to a some constants and variables

$$\tan \delta = \frac{-gk^2}{\mu_0\sigma_s(\omega - kV)}$$

Using the fact that $\omega = \frac{\pi}{\tau} V_s = kV_s$, we can simplify this equation to get

$$\delta = \tan^{-1} \left( \frac{-g(\frac{\pi}{\tau})}{\mu_0\sigma_s(V_s - V)} \right)$$  (14)

Next, looking at the imaginary terms in equation (13),

$$-gk^2H_o i \sin \delta = ikK_o - \mu_0\sigma_s i\omega H_0 \cos \delta + \mu_0\sigma_s ikV H_0 \cos \delta$$

Cancelling the $i$ in all terms and bringing terms with $H_0$ to one side obtains

$$H_0(\mu_0\sigma_s \omega \cos \delta - \mu_0\sigma_s kV \cos \delta - gk^2 \sin \delta) = kK_0$$

Solving for $H_0$ gives

$$H_0 = \frac{kK_0}{\mu_0\sigma_s \cos \delta(\omega - kV) - gk^2 \sin \delta}$$

Using the fact that $\omega = \frac{\pi}{\tau} V_s = kV_s$, a $k$ cancels out of each term and the final equation becomes

$$H_o = \frac{K_o}{\mu_0\sigma_s(V_s - V) \cos \delta - g(\frac{\pi}{\tau}) \sin \delta}.$$  (15)

3 ACCELERATION DUE TO LAUNCH SYSTEMS
Equation (15) represents the magnetic field of one side of the DSLIM’s magnetic field. In electricity and magnetism, Poynting’s Theorem relates the rate of energy transfer (per unit volume) to the rate of work done on a charge distribution plus the energy flux leaving that region. The Poynting vector is used to demonstrate the energy flux density of an electromagnetic field and is expressed by \( \vec{S} = \vec{E} \times \vec{B} \). The units of this vector are watts per square meter. The time average Poynting vector is given as \( \vec{S} = \frac{1}{2} Re(\vec{E} \times \vec{B}^*) \), where the * represents the complex conjugate. This theorem can be used to find the time average thrust of the electromagnetic field over the length of the secondary [Johnson, 2005].

\[
\text{Thrust} = \frac{D}{2} K_1 B_y \cos(-\delta) \int_0^L 1 dx
\]
\[
= \frac{D}{2} L K_1 B_y \cos(-\delta)
\]  

(16)

All parameter values for a short primary double sided LIM motor are taken from Yamamura’s “Linear Induction Motor Theory”[Yamamura, 1979]:

\[
f = 50Hz \quad g = 15mm \quad K_0 = 65300 \frac{A}{m} \quad V_s = 9 \frac{m}{s}
\]
\[
d = 5mm \quad \tau = 90mm \quad \text{poles}=4 \quad D = 90mm \quad \sigma_s = 1.75 \cdot 10^5 S
\]

\( B_y \) is easily obtained from the \( H_0 \) equation with the knowledge that \( B = \mu_0 H \). A factor of two is also added to compensate for both magnetic fields since this is a double sided LIM.

\[
B_y = \frac{2\mu_0 K_o}{\mu_o \sigma_o (V_s - V) \cos \delta - g \left( \frac{\pi}{2} \right) \sin \delta}
\]  

(17)

Using Python in Jupyter Notebook, we can graph the thrust vs. slip. First graphing the thrust vs. slip in Figure 16(a) for a DSLIM with a synchronous speed of 10 \( \frac{m}{s} \) it is observable that the thrust is zero at zero slip conditions. It then shoots up at around 0.2-0.3 slip and falls back down when the slip approaches 1. Figure 16(b) shows the thrust vs. slip of a DSLIM with a synchronous speed of 40 \( \frac{m}{s} \). For the Figure 16(b), the thrust is efficient in a smaller range of slip than Figure 16(b). This uncovers an efficiency issue of DSLIMs. While these launch systems do work, they are limited to smaller velocities if they are to be more efficient.
Next, the thrust vs. slip is graphed for two different airgap lengths. Figure 17(a) shows the thrust vs. slip of a DSLIM with an airgap in which the secondary glides through of 10mm. Figure 17(b) shows the thrust vs. slip of the same DSLIM, but with an airgap of 15mm. Looking at these two figures concludes that when a DSLIM has a smaller airgap, the thrust output is much higher. Though it is very tricky to make the airgap much smaller because if the primary and secondary get too close and interact, it could be detrimental to the DSLIM hardware.
4 Discussion

Roller coasters incorporate many different areas of physics in order to be studied and understood. In classical physics, the laws of energy, Newton’s laws, and the conservation of energy can be used to study perfectly circular loops. Most general physics 1 and 2 college courses are limited to this type of physics. It is a good way to understand basic principles and ideas before moving to more complex ones. Next, applying concepts of differential equations and high levels of calculus, roller coaster loops of different shapes can be studied. These shapes are more realistic than circular ones as a result of safety concerns. The concept of these curved entry teardrop shaped loops can be applied to areas other than coasters as well. For example, flying jet planes. Pilots would have to gradually decrease the radius of curvature to avoid harsh sudden changes of G forces when completing loops.

Zooming in to the mechanics of roller coasters, launch systems such as linear induction motors can be studied using electricity and magnetism. Poynting’s Theorem can be used to model the power output of these launch systems. Many different scenarios can be modeled by playing with different parameters in programs such as Python. LIMs have applications outside of the coaster world as well. They are often used to propel trains, launch aircraft, and as lifting mechanisms such as cranes.

Launch systems on coasters have expanded the realm of coaster building. They are useful when design teams have limited space to work with. Launch systems will get the train to a sufficient speed without the need of a lift hill, which can take up more space. Launches can also be seen as more thrilling. Some parks will build a launch coaster to draw in the thrill seekers. LIMs do have their downsides such as being less efficient at higher velocities and losing magnetic flux due to end effect and the airgap. Linear induction motors are just one of multiple launch systems a park can use for their coasters. Different launch systems all have their pros and cons but LIMs are widely used when it comes to choosing a launch system.
References


Liasi, S. (2015). What are linear motors?


5 Appendix

5.1 Python code for G forces felt on Flip Flap

In [1]: import matplotlib.pyplot as plt
import numpy as np
In [2]: v_0 = 40
g = 9.8
r = 12.5
In [3]: x = np.linspace(0,6,1000)
y = v_0**2/(g*r) + 3*np.cos(x) -2
In [4]: plt.plot(x,y)
plt.xlabel("Radians")
plt.ylabel("Number of G's Felt")
plt.title("Flip Flap Loop G Force Graph")
Out[4]: Text(0.5, 1.0, 'Flip Flap Loop G Force Graph')
5.2 Python code for graphing a loop with constant centripetal acceleration

```python
In [1]: import numpy as np
from matplotlib import pyplot as plt

In [2]: g=9.8
C=2
v_0=20
def xdot(theta):
    return g*np.cos(theta)
def ydot(theta):
    return g*np.sin(theta)
def xdot2(y):
    return C/(((v_0**2)/g)-2*y)

In [3]: h=0.01

In [4]: N=5000
xlist=np.empty(N)
ylist=np.empty(N)
theta_list=np.empty(N)
xlist[0]=0
ylist[0]=0
theta_list[0]=0

In [5]: for i in range(N):
    k1x=h*xdot(theta_list[i])
    k1y=h*ydot(theta_list[i])
    k1t=h*xdot2(ylist[i])
    k2x=h*xdot(theta_list[i]+(k1t/2))
    k2y=h*ydot(theta_list[i]+(k1t/2))
    k2t=h*xdot2(ylist[i]+(k1y/2))
    k3x=h*xdot(theta_list[i]+(k2t/2))
    k3y=h*ydot(theta_list[i]+(k2t/2))
    k3t=h*xdot2(ylist[i]+(k2y/2))
    k4x=h*xdot(theta_list[i]+k3t)
    k4y=h*ydot(theta_list[i]+k3t)
    k4t=h*xdot2(ylist[i]+k3y)
    xlist[i]=xlist[i-1]+k1x/6+k2x/3+k3x/3+k4x/6
    ylist[i]=ylist[i-1]+k1y/6+k2y/3+k3y/3+k4y/6
    theta_list[i]=theta_list[i-1]+k1t/6+k2t/3+k3t/3+k4t/6

In [7]: plt.plot(xlist,ylist)
plt.xlabel('meters')
plt.ylabel('meters')
plt.title('Roller Coaster Loop with a Constant Centripetal Acceleration of 2 G’s')
plt.axis([0,25,0,20])

Out[7]: (0.0, 25.0, 0.0, 20.0)
```

![Roller Coaster Loop with a Constant Centripetal Acceleration of 7 Gs](image)

In [ ]:
5.3 First 20 G force calculations in Microsoft Excel for a loop with constant centripetal

\[ G = \frac{\nu_0^2 - 2gh}{rg} + \cos \theta \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( \theta )</th>
<th>r</th>
<th>G's Felt</th>
<th>( \Delta s )</th>
<th>( 0.1 )</th>
</tr>
</thead>
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<td></td>
</tr>
</tbody>
</table>

\( \nu_0 \) = 20 m/s
5.4 Python code for graphing a loop with a constant G force

```python
In [1]: import numpy as np
from matplotlib import pyplot as plt
In [2]: g=9.8
G=3.5
v_0=20
def xdot(theta):
    ... Coaster Loop with a Constant G Force of 3.5")
plt.axis([0,25,0,20])
In [8]:
Out[8]: (0.0, 25.0, 0.0, 20.0)
```

```
In [3]: h=0.01
In [4]: xlist=np.empty(N)
ylist=np.empty(N)
thetalist=np.empty(N)
xlist[0]=0
thetalist[0]=0
In [5]: for i in range(1,N):
    k1x=h*xdot(thetalist[i-1])
    k1y=h*ydot(thetalist[i-1])
    k2x=h*xdot(thetalist[i-1]+(k1x/2))
    k2y=h*ydot(thetalist[i-1]+(k1y/2))
    k3x=h*xdot(thetalist[i-1]+(k2x/2))
    k3y=h*ydot(thetalist[i-1]+(k2y/2))
    k4x=h*xdot(thetalist[i-1]+k3x)
    k4y=h*ydot(thetalist[i-1]+k3y)
    xlist[i]=xlist[i-1]+h*(k1x/6+k2x/3+k3x/3+k4x/6)
    ylist[i]=ylist[i-1]+h*(k1y/6+k2y/3+k3y/3+k4y/6)
    thetalist[i]=thetalist[i-1]+h*(k1t/6+k2t/3+k3t/3+k4t/6)
In [8]: plt.plot(xlist,ylist)
plt.xlabel("meters")
plt.ylabel("meters")
plt.title("Roller Coaster Loop with a Constant G Force of 3.5")
plt.axis([0,25,0,20])
Out[8]: (0.0, 25.0, 0.0, 20.0)
```

![Roller Coaster Loop with a Constant G Force of 3.5](image)

5 APPENDIX 22
### 5.5 First 20 G force calculations in Microsoft Excel for a loop with a constant G force

\[
G = \frac{v_0^2}{rg} - \frac{2gh}{rg} + \cos \theta
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( \theta )</th>
<th>r</th>
<th>G’s</th>
<th>G</th>
<th>3.5</th>
</tr>
</thead>
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<td>( v_0 )</td>
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<td>( \Delta s )</td>
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</table>
5.6 Python code for a DSLIM with a synchronous speed of $10 \frac{m}{s}$

```python
In [1]: import matplotlib.pyplot as plt
import numpy as np

In [2]: slipL = np.empty(100)
   for i in range(100):
       slipL[i] = i * 0.01

In [3]: deltaL = np.empty(100)
   B_yL = np.empty(100)
   thrustL = np.empty(100)
   VL = np.empty(100)

In [4]: f = 50
   K1 = 65300
   V_s = 10
   tau = 0.09
   D = 0.09
   mu0 = 1.256 * 10**(-6)
   sigma_s = 1.75 * 10**(-5)
   g = 0.015
   L = 6
   for i in range(100):
       VL[i] = (1 - slipL[i]) * V_s

In [5]: def delta(VL):
       return np.arctan((np.pi) / (tau * mu0 * sigma_s * (V_s - VL)))

In [6]: def B_y(delta, VL):
       return (2 * mu0 * K1) / (g * np.pi * np.sin(delta) * sigma_s * mu0 * (V_s - VL) * np.cos(delta))

In [7]: def thrust(B_y, delta):
       return (D / 2) * L * K1 * B_y * np.cos(-delta)

In [8]: for i in range(100):
   deltaL[i] = delta(VL[i])
   B_yL[i] = B_y(deltaL[i], VL[i])
   thrustL[i] = thrust(B_yL[i], deltaL[i])

In [10]: plt.plot(slipL, thrustL)
plt.xlabel('Slip')
plt.ylabel('Thrust(N)')
plt.title('Thrust vs. Slip for v=10 m/s')
plt.show()
```

![Graph showing thrust vs. slip for a speed of 10 m/s](image.jpg)
5.7 Python code for a DSLIM with a synchronous speed of $40 \text{ m/s}$

```python
In [1]: import matplotlib.pyplot as plt
   import numpy as np

In [2]: slipL = np.empty(100)
   for i in range(100):
       slipL[i] = 0.01

In [3]: deltaL = np.empty(100)
   B_yL = np.empty(100)
   thrustL = np.empty(100)
   VL = np.empty(100)

In [4]: f = 50
   K1 = 65300
   V_s = 40
   tau = 0.09
   D = 0.09
   mu0 = 1.256 * 10**(-6)
   sigma_s = 1.75 * 10**(-5)
   g = 0.015
   L = 6
   for i in range(100):
       VL[i] = (1 - slipL[i]) * V_s

In [5]: def delta(VL):
   return np.arctan(g * np.pi / (tau * mu0 * sigma_s * (V_s - VL)))

In [6]: def B_y(delta, VL):
   return (2 * mu0 * K1) / (g * np.pi / tau * np.sin(delta) * sigma_s * mu0 * (V_s - VL) * np.cos(delta))

In [7]: def thrust(B_y, delta):
   return (D / 2) * X * B_y * np.cos(-delta)

In [8]: for i in range(100):
   deltaL[i] = delta(VL[i])
   B_yL[i] = B_y(deltaL[i], VL[i])
   thrustL[i] = thrust(B_yL[i], deltaL[i])

In [9]: plt.plot(slipL, thrustL)
   plt.xlabel("Slip")
   plt.ylabel("Thrust(N)")
   plt.title("Thrust vs. Slip for $v=40$ m/s")
   plt.show()
```

![Thrust vs. Slip for $v=40$ m/s](image)

5 APPENDIX
5.8 Python code for a DSLIM with an airgap of 10mm

```python
In [2]: importmatplotlib.pyplot asplt
importnumpy asnp

In [3]: slipL = np.empty(100)
   for i in range(100):
       slipL[i]=i*.01

In [4]: deltaL = np.empty(100)
   B_yL = np.empty(100)
   thrustL = np.empty(100)
   VL = np.empty(100)

In [5]: f = 50
   K1 = 65300
   V_s = 10
   tau = 0.09
   D = 0.09
   mu0 = 1.256 * 10**(6)
   sigma_s = 1.75 * 10**(5)
   g = 0.01
   L = 1
   for i in range(100):
       VL[i] = (1-slipL[i])*V_s

In [6]: def delta(VL):
   return np.arctan(g*(np.pi)/(tau*mu0*sigma_s*(V_s-VL)))

In [7]: def B_y(delta, VL):
   return (2*mu0*K1)/(g*(np.pi/tau))*np.sin(delta)*sigma_s*mu0*(V_s-VL)*np.cos(delta)

In [8]: def thrust(B_y, delta):
   return (D/2)*V1*B_y*np.cos(-delta)

In [9]: for i in range(100):
   deltaL[i] = delta(VL[i])
   B_yL[i] = B_y(deltaL[i], VL[i])
   thrustL[i] = thrust(B_yL[i], deltaL[i])

In [10]: plt.plot(slipL, thrustL)
plt.xlabel('Slip')
plt.ylabel('Thrust(N)')
plt.title('Thrust vs. Slip for Airgap=10mm')
plt.show()
```

---

The code above calculates the thrust, 

\[ T = \frac{D}{2} \times V1 \times B_y \times \cos(-\delta) \]

for a DSLIM with an airgap of 10mm. The thrust is plotted against the slip for each value of the slip, which is calculated using the formula:

\[ \delta = \arctan\left(\frac{g \times \pi}{\tau \times \mu_0 \times \sigma_s \times (V_s - V_L)}\right) \]

where:
- \( g \) is the gravitational constant.
- \( \tau \) is the mean time constant.
- \( \mu_0 \) is the permeability of free space.
- \( \sigma_s \) is the saturation conductivity.
- \( V_s \) is the source voltage.
- \( V_L \) is the load voltage.
- \( D \) is the diameter of the shaft.
- \( L \) is the length of the stator.

The code also plots the thrust against the slip, showing how the thrust varies with the slip for an airgap of 10mm.
5.9 Python code for a DSLIM with an airgap of 15mm

```python
In [1]: import matplotlib.pyplot as plt
import numpy as np
In [2]: slipL = np.empty(100)
for i in range(100):
    slipL[i] = i*.01
In [3]: deltaL = np.empty(100)
B_yL = np.empty(100)
thrustL = np.empty(100)
VL = np.empty(100)
In [4]: f = 50
K1 = 65300
V_s = 10
tau = 0.09
D = 0.09
mu0 = 1.256 * 10**(-6)
sigma_s = 1.75 * 10**(-6)
g = 0.015
L = 6
for i in range(100):
    VL[i] = (1-slipL[i])*V_s
In [5]: def delta(VL):
    return np.arctan(g*(np.pi)/(tau*mu0*sigma_s*(V_s - VL)))
In [6]: def B_y(delta, VL):
    return (2*mu0*K1)/(g*(np.pi/tau)*np.sin(delta)*sigma_s*mu0*(V_s-VL))*np.cos(delta)
In [7]: def thrust(B_y, delta):
    return (D/2)*L*K1*B_y*np.cos(-delta)
In [8]: for i in range(100):
    deltaL[i] = delta(VL[i])
    B_yL[i] = B_y(deltaL[i], VL[i])
    thrustL[i] = thrust(B_yL[i], deltaL[i])
In [10]: plt.plot(slipL, thrustL)
plt.xlabel('Slip')
plt.ylabel('Thrust(N)')
plt.title('Thrust vs. Slip for Airgap=15mm')
plt.show()
```

![Thrust vs Slip for Airgap=15mm](image)