Undergraduate Review

Volume 6 Article 21

2010

A Two-Dimensional, Linear–Elastic Model to Explain Radial Extensional Fractures, Pantheon Fossae, Caloris Basin, Mercury

Brianne McDonough

A Two-Dimensional, Linear–Elastic Model to Explain Radial Extensional Fractures, Pantheon Fossae, Caloris Basin, Mercury

Brianne McDonough

Brianne McDonough is a graduating Earth Science major with a concentration in Geology. Through the Adrian Tinsley Program, Brianne completed her project titled, *A Two-Dimensional, Linear-Elastic Model to Explain Radial Extensional Fractures, Pantheon Fossae, Caloris Basin, Mercury,* under the guidance of Dr. Robert Cicerone. Brianne is currently looking into employment within the realm of higher education and will eventually pursue a Master's degree in Remote Sensing.

In this study, two-dimensional linear elasticity theory is used to model the lithospheric stress field that produces radial extensional fractures observed at Pantheon Fossae in the Caloris Basin of Mercury. These fractures were imaged by the MESSENGER mission flyby of Mercury on January 14, 2008 and show radial fractures extending outward from a 40-kilometer impact crater named Apollodorus. Recent studies have proposed several different mechanisms to explain these fractures, including magmatic processes, central basin uplift, and stresses produced by the formation of the impact crater itself.

The model presented here attempts to describe the state of the stress field, independent of the physical mechanism that produced it. The first part of the analysis uses a model with azimuthal symmetry, consisting of a two-dimensional infinite plate with a hole in the center to represent the crater and a constant horizontal pressure applied along the crater wall. This model produces a stress field with compressive stresses in the radial direction and tensional stresses in the azimuthal direction, which is consistent with the formation of radial extensional fractures. However, this simple model cannot explain the observed asymmetry of the fractures distribution, where fractures extend further and are more abundant along a preferred azimuth of approximately N30°E. The second part of the analysis superposes an anisotropic regional stress field, with the maximum horizontal compressive stress aligned with this direction of maximum fracture extent. This analysis shows that the yield strength of the lithosphere is minimal along the direction of the maximum compressive stress. Therefore, a stress field with constant pressure applied horizontally along the crater wall superimposed upon a regional stress field with maximum horizontal compressive stress aligned along a N30°E azimuth can adequately explain the observed fracture distribution.

Introduction

The MESSENGER (MErcury Surface, Space ENvironment, GEochemistry, and Ranging) mission was launched by NASA on August 3, 2004. As part of its complicated trajectory to successfully enter the planet's orbit, the spacecraft performed one flyby of Earth on August 2, 2005, two flybys of Venus on October 24, 2006 and June 5, 2007, and three flybys of Mercury on January 14, 2008, October 6, 2008, September 29, 2009. The spacecraft will finally enter orbit around Mercury on March 18, 2011. This study will focus on the January 14, 2008 flyby. This flyby allowed images to be taken of parts

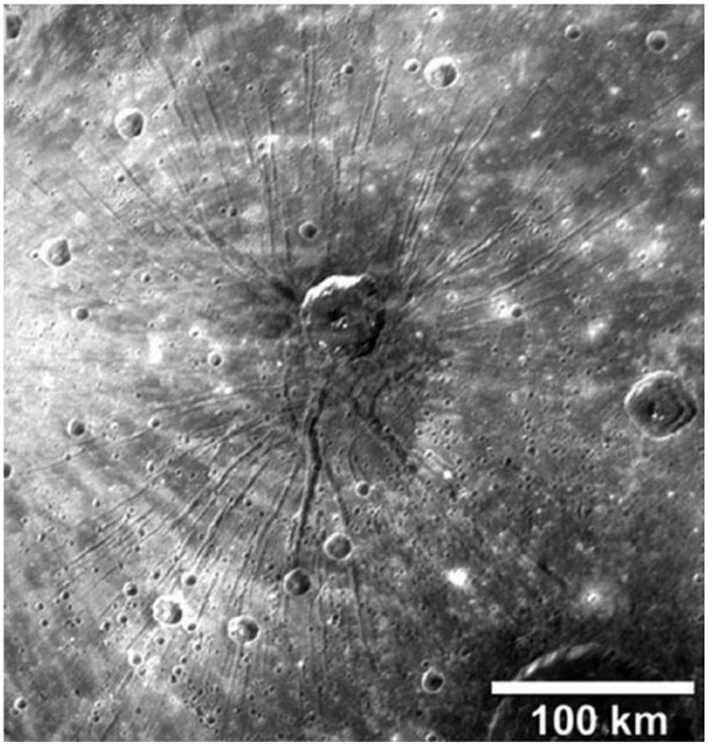

Figure 1. MESSENGER image of Pantheon Fossae, Caloris Basin, Mercury taken on the January 14, 2008 flyby of the planet (MESSENGER MDIS NAC image EN0108828540M, centered at 29.9°N, 162.9°E). Extensional radial fractures are evident, extending outward from the central crater Apollodorus.

of the planet that were not imaged by the only other mission to visit Mercury, namely Mariner 10, in 1974. One of the more striking images taken was of an area in the center of the Caloris Basin (a large impact feature on the surface of Mercury) named Pantheon Fossae (Figure 1) by a flyby of the planet on January 14, 2008. This image shows an impact crater called Apollodorus, approximately 40 kilometers in diameter, which lies in the center of a radial pattern of extensional fractures. The unusual pattern of fractures inspired NASA personnel to nickname the feature the "Spider Crater".

Extensional fractures are known to be caused by tensional stresses (i.e., "pull-apart" stresses), yet the conventional wisdom is that stresses in Mercury's lithosphere (i.e., surface layer) should be compressional (i.e., "squeeze-together" stresses). These compressional stresses are hypothesized to be the result of a shrinking, or volume reduction, of Mercury of about 5% early in the planet's history (Melosh and McKinnon, 1988). The Pantheon Fossae region, and in fact the Caloris Basin in general, are somewhat enigmatic in that they show evidence of tensional stress fields due to the presence of these extensional fractures.

Several different mechanisms have been proposed to explain the observed fracture distribution at Pantheon Fossae. Head et al. (2008) have attributed the origin of the radial fractures and dikes to molten material (magma) rising from the interior of the planet, producing horizontally-directed pressure around an approximately circular region at the center of the fracture pattern. This horizontally-directed pressure is produced by the rising magma, inducing "pull-apart" stresses that can cause the fractures to develop. Murchie et al. (2008) present a model where horizontally-directed pressure is produced not by rising magma, but by deformation of the lithosphere (outer layer) of the planet due to lateral flow of material from outside the area of the basin, producing uplift in the basin interior and subsequently causing "pull-apart" stresses to develop, thereby causing the fractures. While the details of this mechanism are different from the magmatic-intrusion mechanism, the basic mechanical model using two-dimensional, linear elasticity is still appropriate, where in this case the horizontally-directed pressure results from the central basin uplift.

Alternatively, Freed et al. (2009) have advocated for an impact-related mechanism for the extensional fractures, where the formation of the Apollodorus crater itself caused a perturbation in the stress field of the Pantheon Fossae region, thereby leading to the formation of the fractures. Again, while the details of this mechanism are also different, the basic mechanical model using two-dimensional, linear elasticity is appropriate to model the horizontally-directed pressure produced by the impact-generated perturbation in the stress field.

Two-Dimensional Linear Elasticity Theory

The method proposed in this study is to use the powerful mathematical tools of linear elasticity theory to calculate stress fields that can be used to model the distribution of fractures observed at Pantheon Fossae. The model presented here attempts to describe the state of the stress field independent of the physical mechanism that produced it.

The theory of linear elasticity involves solving the so-called biharmonic equation

$$\nabla^4 \Phi = 0,$$

where Φ is called the Airy stress function. Using a polar coordinate system with circular symmetry to simplify the analysis, the radial stress σ_{rr}, circumferential stress $\sigma_{\theta\theta}$, and shear stress $\sigma_{r\theta}$ are given by

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2},$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \Phi}{\partial r^2},$$

$$\sigma_{r\theta} = \frac{1}{r^2}\frac{\partial \Phi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \Phi}{\partial r \partial \theta}$$

where r is the distance from the origin of the coordinate system and θ is the angle measured counterclockwise from the positive x-axis. The geometry of the problem is summarized in Figure 2.

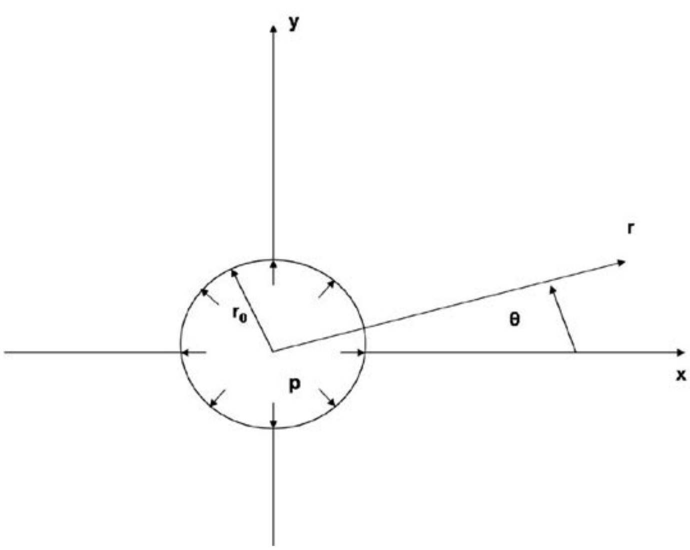

Figure 2. Polar coordinate system (r,θ) geometry for the two-dimensional, linear elastic model relative to a Cartesian (x-y) coordinate system. The coordinate r is the distance (radius) from the origin, θ is the angle of the radius vector with respect to the x-axis, p is the central pressure in the hole, and r_0 is the radius of the hole.

The first part of the analysis uses a model with azimuthal symmetry, consisting of a two-dimensional infinite plate to model the lithosphere (outer layer) of Mercury with a hole in the center to represent the crater and a constant horizontal pressure applied along the crater wall (Figure 2). In this case, variations in the stress fields will depend only on the distance r from the crater wall, and all derivatives with respect to θ are equal to zero. Therefore, the stress equations reduce to the simple form

$$\sigma_{rr} = p\left(\frac{r_0}{r}\right)^2,$$

$$\sigma_{\theta\theta} = -p\left(\frac{r_0}{r}\right)^2,$$

$$\sigma_{r\theta} = 0,$$

where p is the central pressure in the hole and r_0 is the radius of the hole.

The normalized compressional and tensional stresses as a function of normalized distance calculated from the above equations are shown in Figure 3. The sign convention used here is the one typically used in geology, i.e., compressional stresses are taken as positive and tensional stresses are taken as negative. The results show that both the compressive and tensional stresses produced in Mercury's lithosphere by a pressure applied to the crater wall are at their maximum

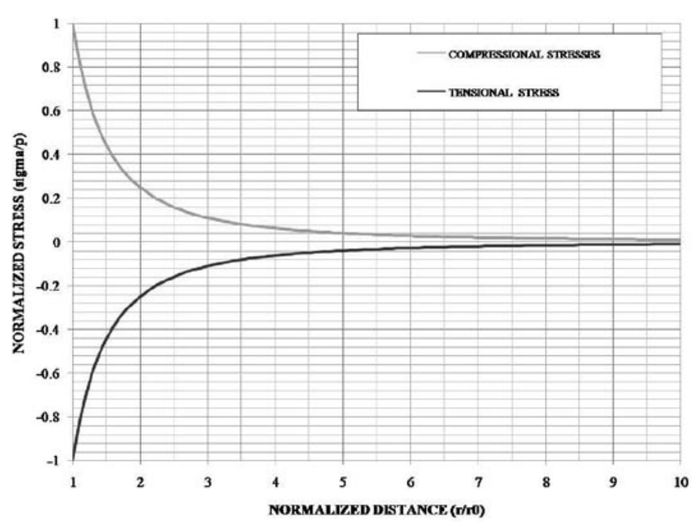

Figure 3. Normalized compressional and tensional stresses as a function of normalized distance from the crater wall for the case of azimuthal symmetry. The stresses are normalized by the pressure at the crater wall and the distance is normalized by the crater radius. The stress field decrease as the square of the distance from the crater wall.

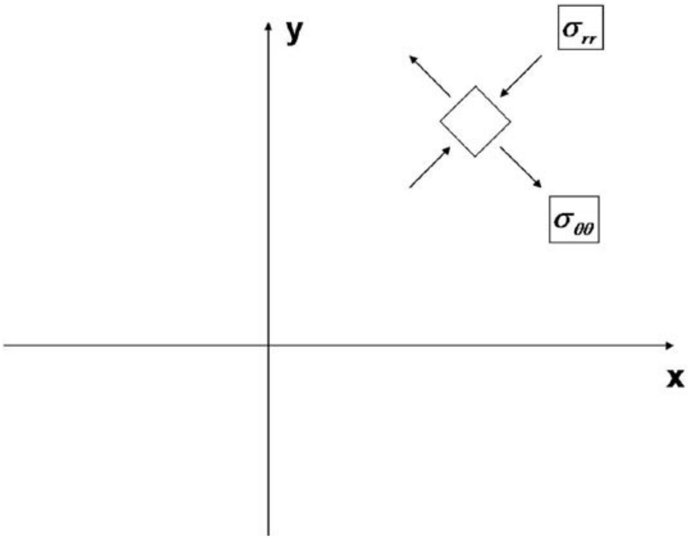

Figure 4. Stress field at an arbitrary point in the medium. The radial stresses are compressional, and the circumferential stresses are tensional, which is consistent with a stress configuration that would produce radial extensional fractures.

values at the crater wall and decrease with the square of the distance from the crater center. Therefore, at an arbitrary point in the medium, the radial stresses are compressive and the circumferential stresses are tensional (Figure 4). This stress condition is consistent with the formation of radial extensional fractures, as extensional fractures are known to form parallel to the direction of maximum compressive stress (e.g., Johnson, 1970).

Fracture Asymmetry and Anisotropic Regional Stresses

The results from a simple azimuthally-symmetric model predict that the distribution of fractures should be uniform around the circumference of the crater wall. However, the image of Pantheon Fossae in Figure 1 clearly shows that the fractures are both more abundant and of longer extent in a preferred direction, namely along an azimuth oriented approximately 30° east of true north. To quantify this fracture distribution, a Rose diagram was constructed, showing the total fracture length as a function of azimuth (Figure 5). This diagram confirms the preferred direction of fracture formation along the N30°E azimuth. Therefore, while the simple azimuthally-symmetric model explains the general features of the fractures in terms of the stress field that produced them, the actual azimuthal asymmetry of the fractures indicates a more complex stress-field configuration than that predicted by this simple model.

Azimuthal asymmetry, as seen is the actual fracture pattern at Pantheon Fossae, can be attributed to an additional regional, or far-field, stress. A similar pattern of fractures has been observed in the Spanish Peaks area of south-central Colorado

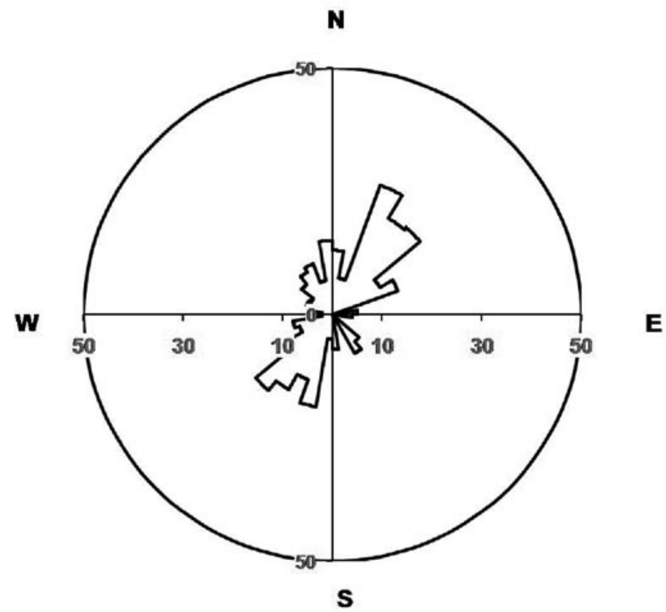

Figure 5. Rose diagram showing the distribution of fractures. The diagram was obtained by determining the total fracture length within azimuthal windows of 10°. Note the preferred direction of fractures along an azimuth of approximately N30°E.

(Odé, 1957) and was explained by the presence of a regional stress field in the area, with the maximum compressive stress aligned with the direction of maximum fracture length. Such a stress field is shown schematically in Figure 6.

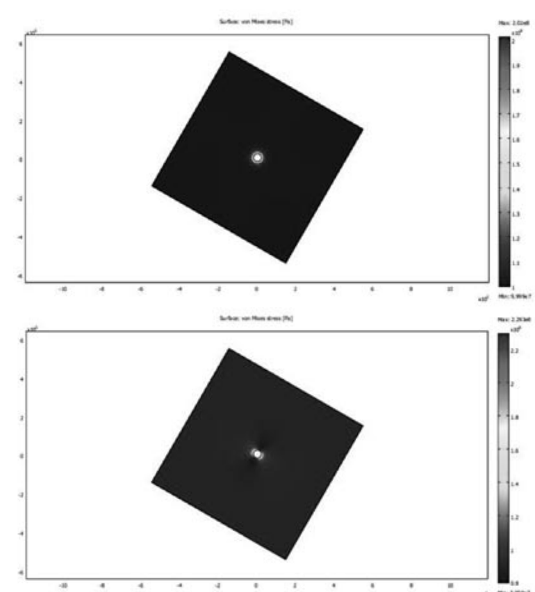

Figure 6. COMSOL Multiphysics modeling results for the regional stress field. The maximum compressive stress S_1 of 100 MPa is applied along an azimuth of N30°E, and the minimum compressive stress S_2 is applied in the perpendicular direction. The minimum compressive stress S_2 has a value of 100 MPa in the upper diagram and 70 MPa in the lower diagram.

Muller and Pollard (1977) have shown that such a stress field can be represented by the Airy stress function Φ_f of the form

$$\Phi_f = \frac{1}{2} S_1 r^2 \cos^2(\theta - \phi) + \frac{1}{2} S_2 r^2 \sin^2(\theta - \phi),$$

where S_1 and S_2 are the maximum and minimum regional stress-field components, respectively and ϕ is the angle between the maximum stress field component and the x-axis, measured counterclockwise. Then the radial stress σ^f_{rr}, circumferential stress $\sigma^f_{\theta\theta}$, and shear stress $\sigma^f_{r\theta}$ produced by the far-field stress are given by

$$\sigma^f_{rr} = S_2 + (S_1 - S_2)\cos^2(\theta - \phi),$$

$$\sigma^f_{\theta\theta} = S_2 + (S_1 - S_2)\sin^2(\theta - \phi),$$

$$\sigma^f_{r\theta} = -(S_1 - S_2)\sin(\theta - \phi)\cos(\theta - \phi).$$

In addition to the azimuthal asymmetry, shear stresses are produced by this additional regional stress field. In this study, the COMSOL Multiphysics software was used to do some preliminary modeling to determine the effect of a superimposed regional stress field. The software determines the so-called von Mises stress, which can be interpreted as a measure of the strength of the medium under the applied stress field.

To replicate the conditions on the surface of Mercury, a square plate of dimension 800 km by 800 km was used, with a hole of 20 km radius at the center to model the crater. The material properties of the plate are those of basalt, the rock that is presumed to constitute the lithosphere of Mercury. An anisotropic (i.e., not equal in all directions) regional stress field was applied, with a maximum compressive stress S_1 of 100 MPa along an azimuth of 30° and the minimum compressive stress S_2 along a perpendicular direction. The maximum compressive stress of 100 Mpa is approximately the order of magnitude of the stress expected in Mercury's lithosphere (Melosh and McKinnon, 1988). The minimum compressive stress was varied over values of 100 and 70 MPa. These results are shown in Figure 6.

The modeling results show that the von Mises stress, and therefore the strength, of the material is weakest along the azimuth of the maximum compressive stress. In other words, if the lithosphere of Mercury is "pre-stressed" by this regional anisotropic stress field, then the application of a constant pressure around the crater would cause the material to yield most easily along this direction first. This is consistent with the observation that the fractures extend furthest along this direction.

Summary
A simple two-dimensional, linear-elastic theory is shown to explain the presence of radial extensional fractures observed at Pantheon Fossae, Mercury. The model uses an infinite plate with a hole to represent the crater, with a constant pressure applied along the wall of the crater. The model produces a stress field with radial compressional stresses and circumferential tensional fractures that decrease with the square of the distance from the crater wall. This stress field is consistent with the formation of radial extensional fractures. The superposition of an anisotropic regional stress field, with a maximum compressive stress aligned along the direction of the maximum fracture extent, is shown to be a likely explanation of the observed asymmetry of the fractures, namely, the existence of a preferred direction of maximum fracture extent. However, additional modeling is needed to better quantify the nature of the regional stress field.

Acknowledgements
I would like to thank the Adrian Tinsley Program (ATP) at Bridewater State College for providing funding in the summer of 2009 to support this research. I would also like to thank Professor Robert Cicerone of the Department of Earth Sciences at Bridgewater State College for serving as my mentor on this project and for providing advice and encouragement.

References

Freed, Andrew M., Sean C. Solomon, Thomas R. Watters, Roger J. Phillips, and Maria T. Zuber (2009). Could Pantheon Fossae be the result of the Apollodorus crater-forming impact within the Caloris Basin, Mercury?, 40th Lunar and Planetary Science Conference.

Head, James W., Scott L. Murchie, Louise M. Prockter, Mark S. Robinson, Sean C. Solomon, Robert G. Strom, Clark R. Chapman, Thomas R. Watters, William E. McClintock, David T. Blewett, and Jeffrey J. Gillis-Davis (2008). Volcanism on Mercury: evidence from the first MESSENGER flyby, *Science*, **321**, 69-72.

Johnson, Arvid M. (1970). *Physical Processes in Geology*, Freeman, Cooper and Company, San Francisco, CA, 577 pp.

Melosh, H.J. and W.B. McKinnon (1988). The tectonics of Mercury, in *Mercury,* edited by Faith Vilas, Clark R. Chapman, and Mildred Shapley Matthews, University of Arizona Press, 374-4900.

Muller, Otto H. and David D. Pollard (1977). The stress state near Spanish Peaks, Colorado determined from a dike pattern, *Pure and Applied Geophysics*, **115**, 69-86.

Murchie, Scott L., Thomas R. Watters, Mark S. Robinson, James W. Head, Robert G. Strom, Clark R. Chapman, Sean C. Solomon, William E. McClintock, Louise M. Prockter, Deborah L. Domingue, and David T. Blewett (2008). Geology of the Caloris Basin, Mercury: a view from MESSENGER, *Science*, **321**, 73-76.

Odé, Helmer (1957). Mechanical analysis of the dike pattern of the Spanish Peaks area, Colorado, *Bulletin of the Geological Society of America*, **68**, 567-576.